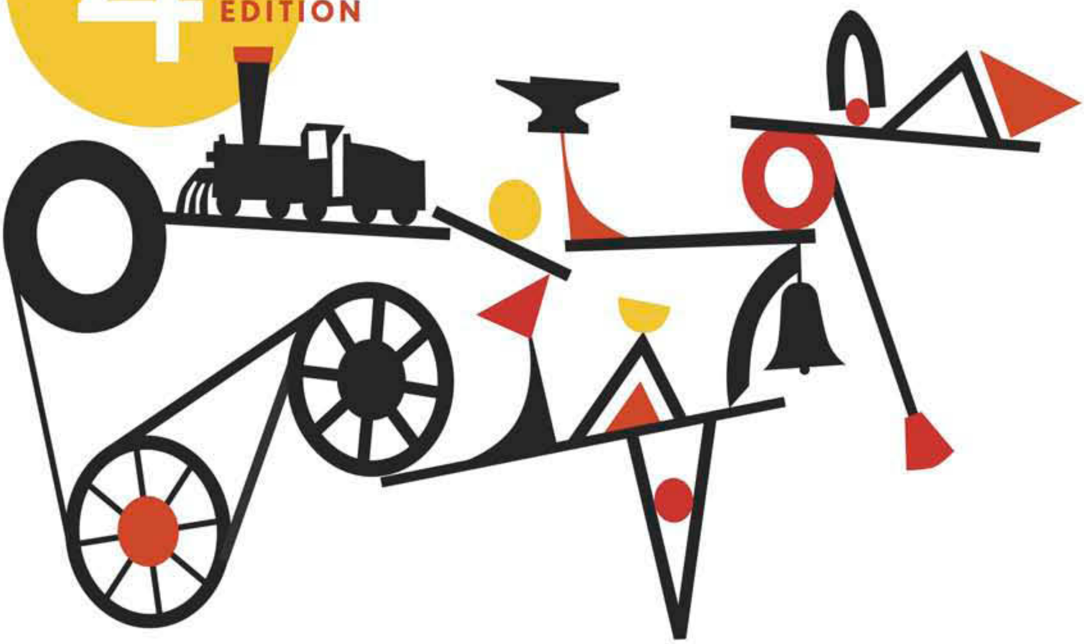
4th EDITION



GAMES OF STRATEGY

DIXIT • SKEATH • REILEY

GAMES OF STRATEGY

FOURTH EDITION

GAMES OF STRATEGY

Fourth Edition

Avinash Dixit
Princeton University

Susan Skeath
Wellesley College

David Reiley
Google

W. W. Norton & Company
New York • London

W. W. Norton & Company has been independent since its founding in 1923, when William Warder Norton and Mary D. Herter Norton first published lectures delivered at the People's Institute, the adult education division of New York City's Cooper Union. The firm soon expanded its program beyond the Institute, publishing books by celebrated academics from America and abroad. By mid-century, the two major pillars of Norton's publishing program—trade books and college texts—were firmly established. In the 1950s, the Norton family transferred control of the company to its employees, and today—with a staff of four hundred and a comparable number of trade, college, and professional titles published each year—W. W. Norton & Company stands as the largest and oldest publishing house owned wholly by its employees.

PRINTED IN THE UNITED STATES OF AMERICA.

Editor: Jack Repcheck
Editorial Assistant: Theresia Kowara
Copyeditor: Christopher Curioli
Project Editor: Sujin Hong
Electronic Media Editor: Carson Russell
Marketing Manager, Economics: Janise Turso
Production Manager: Sean Mintus
Text Design: Jack Meserole
Composition: A-R Editions
Manufacturing: Courier Kendallville

Library of Congress Cataloging-in-Publication Data
Dixit, Avinash K.
Games of strategy / Avinash Dixit, Susan Skeath, David Reiley.—Fourth edition.
pages cm
Includes bibliographical references and index.

ISBN 978-0-393-91968-4 (hardcover)

1. Game theory. 2. Policy sciences. 3. Decision making. I. Skeath, Susan. II. Reiley, David. III. Title.
HB144.D59 2015
519.3—dc23 2014037581

W. W. Norton & Company, Inc., 500 Fifth Avenue, New York, N.Y. 10110
www.wwnorton.com

W. W. Norton & Company Ltd., Castle House, 75/76 Wells Street, London W1T 3QT

1 2 3 4 5 6 7 8 9 0

To the memory of my father,

Kamalakar Ramachandra Dixit

— Avinash Dixit

To the memory of my father,

James Edward Skeath

— Susan Skeath

To my mother,

Ronie Reiley

— David Reiley

Contents

Preface xx

PART ONE
Introduction and General Principles

1 Basic Ideas and Examples 3

1 WHAT IS A GAME OF STRATEGY? 4

2 SOME EXAMPLES AND STORIES OF STRATEGIC GAMES 6

A. Which Passing Shot? 6

B. The GPA Rat Race 7

C. "We Can't Take the Exam Because We Had a Flat Tire" 9

D. Why Are Professors So Mean? 10

E. Roommates and Families on the Brink 11

F. The Dating Game 13

3 OUR STRATEGY FOR STUDYING GAMES OF STRATEGY 14

2 How to Think about Strategic Games 17

1 DECISIONS VERSUS GAMES 18

2 CLASSIFYING GAMES 20

A. Are the Moves in the Game Sequential or Simultaneous? 20

B. Are the Players' Interests in Total Conflict or Is There Some Commonality? 21

C. Is the Game Played Once or Repeatedly, and with the Same or Changing Opponents? 22

D. Do the Players Have Full or Equal Information? 23

E. Are the Rules of the Game Fixed or Manipulable? 25

F. Are Agreements to Cooperate Enforceable? 26

3 SOME TERMINOLOGY AND BACKGROUND ASSUMPTIONS 27

A. Strategies 27

B. Payoffs 28

C. Rationality 29

D. Common Knowledge of Rules 31

E. Equilibrium 32

F. Dynamics and Evolutionary Games 34

G. Observation and Experiment 35

4 THE USES OF GAME THEORY 36

5 THE STRUCTURE OF THE CHAPTERS TO FOLLOW 38

SUMMARY 41

KEY TERMS 41

EXERCISES 42

PART TWO

Concepts and Techniques

3 Games with Sequential Moves 47

1 GAME TREES 48

A. Nodes, Branches, and Paths of Play 48

B. Uncertainty and "Nature's Moves" 48

C. Outcomes and Payoffs 50

D. Strategies 50

E. Tree Construction 51

2 SOLVING GAMES BY USING TREES 52

3 ADDING MORE PLAYERS 57

4 ORDER ADVANTAGES 62

5 ADDING MORE MOVES 63

A. Tic-Tac-Toe 63

B. Chess 65

C. Checkers 69

6 EVIDENCE CONCERNING ROLLBACK 71

7 STRATEGIES IN *SURVIVOR* 75

SUMMARY 80

KEY TERMS 81

EXERCISES 81

4 Simultaneous-Move Games: Discrete Strategies 91

1 DEPICTING SIMULTANEOUS-MOVE GAMES WITH DISCRETE STRATEGIES 92

2 NASH EQUILIBRIUM 94

A. Some Further Explanation of the Concept of Nash Equilibrium 95

B. Nash Equilibrium as a System of Beliefs and Choices 97

3 DOMINANCE 99

A. Both Players Have Dominant Strategies 100

B. One Player Has a Dominant Strategy 101

C. Successive Elimination of Dominated Strategies 104

4 BEST-RESPONSE ANALYSIS 106

5 THREE PLAYERS 108

6 MULTIPLE EQUILIBRIA IN PURE STRATEGIES 111

A. Will Harry Meet Sally? Pure Coordination 111

B. Will Harry Meet Sally? And Where? Assurance 113

C. Will Harry Meet Sally? And Where? Battle of the Sexes 114

D. Will James Meet Dean? Chicken 116

7 NO EQUILIBRIUM IN PURE STRATEGIES 118

SUMMARY 120

KEY TERMS 120

EXERCISES 121

5 Simultaneous-Move Games: Continuous Strategies, Discussion, and Evidence 133

1 PURE STRATEGIES THAT ARE CONTINUOUS VARIABLES 134

A. Price Competition 134

B. Some Economics of Oligopoly 138

C. Political Campaign Advertising 139

D. General Method for Finding Nash Equilibria 142

2 CRITICAL DISCUSSION OF THE NASH EQUILIBRIUM CONCEPT 143

A. The Treatment of Risk in Nash Equilibrium 144

B. Multiplicity of Nash Equilibria 146

C. Requirements of Rationality for Nash Equilibrium 148

3 RATIONALIZABILITY 149

A. Applying the Concept of Rationalizability 150

B. Rationalizability Can Take Us All the Way to Nash Equilibrium 152

4 EMPIRICAL EVIDENCE CONCERNING NASH EQUILIBRIUM 155

A. Laboratory Experiments 156

B. Real-World Evidence 161

SUMMARY 165

KEY TERMS 166

EXERCISES 166

APPENDIX: Finding a Value to Maximize a Function 176

6 Combining Sequential and Simultaneous Moves 180

1 GAMES WITH BOTH SIMULTANEOUS AND SEQUENTIAL MOVES 181

A. Two-Stage Games and Subgames 181

B. Configurations of Multistage Games 185

2 CHANGING THE ORDER OF MOVES IN A GAME 187

A. Changing Simultaneous-Move Games into Sequential-Move Games 188

B. Other Changes in the Order of Moves 193

3 CHANGE IN THE METHOD OF ANALYSIS 194

A. Illustrating Simultaneous-Move Games by Using Trees 194

B. Showing and Analyzing Sequential-Move Games in Strategic Form 196

4 THREE-PLAYER GAMES 200

SUMMARY 203

KEY TERMS 204

EXERCISES 204

7 Simultaneous-Move Games: Mixed Strategies 214

1 WHAT IS A MIXED STRATEGY? 215

2 MIXING MOVES 216

A. The Benefit of Mixing 216

B. Best Responses and Equilibrium 218

3 NASH EQUILIBRIUM AS A SYSTEM OF BELIEFS AND RESPONSES 221

4 MIXING IN NON-ZERO-SUM GAMES 222

A. Will Harry Meet Sally? Assurance, Pure Coordination, and Battle of the Sexes 223

B. Will James Meet Dean? Chicken 226

5 GENERAL DISCUSSION OF MIXED-STRATEGY EQUILIBRIA 227

A. Weak Sense of Equilibrium 227

B. Counterintuitive Changes in Mixture Probabilities in Zero-Sum Games 228

C. Risky and Safe Choices in Zero-Sum Games 230

6 MIXING WHEN ONE PLAYER HAS THREE OR MORE PURE STRATEGIES 233

A. A General Case 233

B. Exceptional Cases 236

7 MIXING WHEN BOTH PLAYERS HAVE THREE STRATEGIES 237

A. Full Mixture of All Strategies 237

B. Equilibrium Mixtures with Some Strategies Unused 239

8 HOW TO USE MIXED STRATEGIES IN PRACTICE 242

9 EVIDENCE ON MIXING 244

A. Zero-Sum Games 244

B. Non-Zero-Sum Games 248

SUMMARY 249

KEY TERMS 249

EXERCISES 250

APPENDIX: Probability and Expected Utility 263

THE BASIC ALGEBRA OF PROBABILITIES 263

A. The Addition Rule 264

B. The Multiplication Rule 265

C. Expected Values 266

SUMMARY 267

KEY TERMS 267

PART THREE

Some Broad Classes of Games and Strategies

8 Uncertainty and Information 271

1 IMPERFECT INFORMATION: DEALING WITH RISK 273

A. Sharing of Risk 273

B. Paying to Reduce Risk 276

C. Manipulating Risk in Contests 277

2 ASYMMETRIC INFORMATION: BASIC IDEAS 279

3 DIRECT COMMUNICATION, OR "CHEAP TALK" 281

A. Perfectly Aligned Interests 282

B. Totally Conflicting Interests 283

C. Partially Aligned Interests 284

D. Formal Analysis of Cheap Talk Games 290

4 ADVERSE SELECTION, SIGNALING, AND SCREENING 294

A. Adverse Selection and Market Failure 294

B. The Market for "Lemons" 295

C. Signaling and Screening: Sample Situations 298

D. Experimental Evidence 303

5 SIGNALING IN THE LABOR MARKET 304

A. Screening to Separate Types 305

B. Pooling of Types 308

C. Many Types 309

6 EQUILIBRIA IN TWO-PLAYER SIGNALING GAMES 310

A. Basic Model and Payoff Structure 311

B. Separating Equilibrium 312

C. Pooling Equilibrium 315
D. Semiseparating Equilibrium 317

SUMMARY 319

KEY TERMS 320

EXERCISES 321

APPENDIX: Risk Attitudes and Bayes' Theorem 335

1 ATTITUDES TOWARD RISK AND EXPECTED UTILITY 335

2 INFERRING PROBABILITIES FROM OBSERVING CONSEQUENCES 338

SUMMARY 341

KEY TERMS 341

9 Strategic Moves 342

1 A CLASSIFICATION OF STRATEGIC MOVES 343
A. Unconditional Strategic Moves 344
B. Conditional Strategic Moves 345

2 CREDIBILITY OF STRATEGIC MOVES 346

3 COMMITMENTS 348

4 THREATS AND PROMISES 352
A. Example of a Threat: U.S.–Japan Trade Relations 353
B. Example of a Promise: The Restaurant Pricing Game 357
C. Example Combining Threat and Promise: Joint U.S.–China Political Action 359

5 SOME ADDITIONAL TOPICS 360
A. When Do Strategic Moves Help? 360
B. Deterrence versus Compellence 361

6 ACQUIRING CREDIBILITY 362
A. Reducing Your Freedom of Action 362
B. Changing Your Payoffs 364

7 COUNTERING YOUR OPPONENT'S STRATEGIC MOVES 368
A. Irrationality 368
B. Cutting Off Communication 368
C. Leaving Escape Routes Open 369
D. Undermining Your Opponent's Motive to Uphold His Reputation 369
E. Salami Tactics 369

SUMMARY 370

KEY TERMS 371

EXERCISES 371

10 The Prisoners' Dilemma and Repeated Games 377

1 THE BASIC GAME (REVIEW) 378

2 SOLUTIONS I: REPETITION 379

A. Finite Repetition 380

B. Infinite Repetition 381

C. Games of Unknown Length 385

D. General Theory 387

3 SOLUTIONS II: PENALTIES AND REWARDS 389

4 SOLUTIONS III: LEADERSHIP 392

5 EXPERIMENTAL EVIDENCE 395

6 REAL-WORLD DILEMMAS 399

A. Evolutionary Biology 399

B. Price Matching 400

C. International Environmental Policy: The Kyoto Protocol 402

SUMMARY 405

KEY TERMS 405

EXERCISES 406

APPENDIX: Infinite Sums 414

11 Collective-Action Games 417

1 COLLECTIVE-ACTION GAMES WITH TWO PLAYERS 418

A. Collective Action as a Prisoners' Dilemma 419

B. Collective Action as Chicken 421

C. Collective Action as Assurance 422

D. Collective Inaction 422

2 COLLECTIVE-ACTION PROBLEMS IN LARGE GROUPS 423

A. Multiplayer Prisoners' Dilemma 425

B. Multiplayer Chicken 427

C. Multiplayer Assurance 429

3 SPILLOVERS, OR EXTERNALITIES 431
A. Commuting and Spillovers 431
B. Spillovers: The General Case 433
C. Commuting Revisited: Negative Externalities 435
D. Positive Spillovers 439
4 A BRIEF HISTORY OF IDEAS 443
A. The Classics 443
B. Modern Approaches and Solutions 444
C. Applications 450
5 "HELP!": A GAME OF CHICKEN WITH MIXED STRATEGIES 454
SUMMARY 458
KEY TERMS 459
EXERCISES 459

12 Evolutionary Games 465
1 THE FRAMEWORK 466
2 PRISONERS' DILEMMA 470
A. The Repeated Prisoners' Dilemma 472
B. Multiple Repetitions 476
C. Comparing the Evolutionary and Rational-Player Models 477
3 CHICKEN 479
4 THE ASSURANCE GAME 482
5 THREE PHENOTYPES IN THE POPULATION 484
A. Testing for ESS 484
B. Dynamics 485
6 THE HAWK–DOVE GAME 488
A. Rational Strategic Choice and Equilibrium 489
B. Evolutionary Stability for $V > C$ 489
C. Evolutionary Stability for $V < C$ 490
D. $V < C$: Stable Polymorphic Population 491
E. $V < C$: Each Player Mixes Strategies 491
F. Some General Theory 493
7 INTERACTIONS BY POPULATION AND ACROSS SPECIES 495
A. Playing the Field 496
B. Interactions across Species 496

8 EVOLUTION OF COOPERATION AND ALTRUISM 499

SUMMARY 503

KEY TERMS 504

EXERCISES 504

13 Mechanism Design 515

1 PRICE DISCRIMINATION 516

2 SOME TERMINOLOGY 521

3 COST-PLUS AND FIXED-PRICE CONTRACTS 522

A. Highway Construction: Full Information 522

B. Highway Construction: Asymmetric Information 524

4 EVIDENCE CONCERNING INFORMATION REVELATION MECHANISMS 527

5 INCENTIVES FOR EFFORT: THE SIMPLEST CASE 529

A. Managerial Supervision 529

B. Insurance Provision 533

6 INCENTIVES FOR EFFORT: EVIDENCE AND EXTENSIONS 537

A. Nonlinear Incentive Schemes 537

B. Incentives in Teams 539

C. Multiple Tasks and Outcomes 540

D. Incentives over Time 541

SUMMARY 543

KEY TERMS 543

EXERCISES 544

PART FOUR

Applications to Specific Strategic Situations

14 Brinkmanship: The Cuban Missile Crisis 559

1 A BRIEF NARRATIVE OF EVENTS 560

2 A SIMPLE GAME-THEORETIC EXPLANATION 567

3 ACCOUNTING FOR ADDITIONAL COMPLEXITIES 569

4 A PROBABILISTIC THREAT 575

5 PRACTICING BRINKMANSHIP 579

SUMMARY 583

KEY TERMS 584

EXERCISES 584

15 Strategy and Voting 589

1 VOTING RULES AND PROCEDURES 590

- A. Binary Methods 591
- B. Plurative Methods 591
- C. Mixed Methods 593

2 VOTING PARADOXES 594

- A. The Condorcet Paradox 595
- B. The Agenda Paradox 596
- C. The Reversal Paradox 597
- D. Change the Voting Method, Change the Outcome 598

3 EVALUATING VOTING SYSTEMS 600

- A. Black's Condition 601
- B. Robustness 602
- C. Intensity Ranking 602

4 STRATEGIC MANIPULATION OF VOTES 604

- A. Plurality Rule 604
- B. Pairwise Voting 606
- C. Strategic Voting with Incomplete Information 609
- D. Scope for Manipulability 612

5 THE MEDIAN VOTER THEOREM 613

- A. Discrete Political Spectrum 614
- B. Continuous Political Spectrum 617

SUMMARY 620

KEY TERMS 620

EXERCISES 621

16 Bidding Strategy and Auction Design 632

1 TYPES OF AUCTIONS 633

A. Auction Rules 633

B. Auction Environments 635

2 THE WINNER'S CURSE 636

3 BIDDING STRATEGIES 639

A. The English Auction 639

B. First-Price, Sealed-Bid, and Dutch Auctions: The Incentive to Shade 639

C. Second-Price, Sealed-Bid Auctions: Vickrey's Truth Serum 640

4 ALL-PAY AUCTIONS 642

5 HOW TO SELL AT AUCTION 645

A. Risk-Neutral Bidders and Independent Estimates 646

B. Risk-Averse Bidders 647

C. Correlated Estimates 648

6 SOME ADDED TWISTS TO CONSIDER 649

A. Multiple Objects 649

B. Defeating the System 651

C. Information Disclosure 652

D. Online Auctions 653

7 ADDITIONAL READING 656

SUMMARY 657

KEY TERMS 657

EXERCISES 658

17 Bargaining 663

1 NASH'S COOPERATIVE SOLUTION 665

A. Numerical Example 665

B. General Theory 666

2 VARIABLE-THREAT BARGAINING 672

3 ALTERNATING-OFFERS MODEL I: TOTAL VALUE DECAYS 674

4 EXPERIMENTAL EVIDENCE 677

5 ALTERNATING-OFFERS MODEL II: IMPATIENCE 680

6 MANIPULATING INFORMATION IN BARGAINING 685

7 BARGAINING WITH MANY PARTIES AND ISSUES 688

A. Multi-Issue Bargaining 688

B. Multiparty Bargaining 690

SUMMARY 690

KEY TERMS 691

EXERCISES 691

Glossary 695

Index 712

Preface

We wrote this textbook to make possible the teaching of game theory to first- or second-year college students at an introductory or "principles" level without requiring any prior knowledge of the fields where game theory is used—economics, political science, evolutionary biology, and so forth—and requiring only minimal high school mathematics. Our aim has succeeded beyond our expectations. Many such courses now exist where none did 20 years ago; indeed, some of these courses have been inspired by our textbook. An even better sign of success is that competitors and imitators are appearing on the market.

However, success does not justify complacency. We have continued to improve the material in each new edition in response to feedback from teachers and students in these courses and from our own experiences of using the book.

For the fourth edition, the main new innovation concerns mixed strategies. In the third edition, we treated this in two chapters on the basis of a distinction between simple and complex topics. Simple topics included the solution and interpretation of mixed-strategy equilibria in two-by-two games; the main complex topic was the general theory of mixing in games with more than two pure strategies, when some of them may go unused in equilibrium. But we found that few teachers used the second of these two chapters. We have now chosen to gather the simple topics and some basic concepts from the more complex topics into just one chapter on mixed strategies (Chapter 7). Some of the omitted material will be available as online appendices for those readers who want to know more about the advanced topics.

We have improved and simplified our treatment of information in games (Chapter 8). We give an expanded exposition and example of cheap talk that clarifies the relationship between the alignment of interest and the possibility of truthful communication. We have moved the treatment of examples of signaling and screening to an earlier section of the chapter than that of the third edition, better to impress upon students the importance of this topic and prepare the ground for the more formal theory to follow.

The games in some applications in later chapters were sufficiently simple that they could be discussed without drawing an explicit game tree or showing a payoff table. But that weakened the connection between earlier methodological chapters and the applications. We have now shown more of the tools of reasoning about the applications explicitly.

We have continued and improved the collection of exercises. As in the third edition, the exercises in each chapter are split into two sets—solved and unsolved. In most cases, these sets run in parallel: for each solved exercise, there is a corresponding unsolved one that presents variation and gives students further practice. The solutions to the solved set for each chapter are available to all readers at **wwnorton.com/studyspace/disciplines/economics.asp**. The solutions to the unsolved set for each chapter will be reserved for instructors who have adopted the textbook. Instructors should contact the publisher about getting access to the instructors' Web site. In each of the solved and unsolved sets, there are two kinds of exercises. Some provide repetition and drill in the techniques developed in the chapter. In others—and in our view those with the most educational value—we take the student step by step through the process of construction of a game-theoretic model to analyze an issue or problem. Such experience, gained in some solved exercises and repeated in corresponding unsolved ones, will best develop the students' skills in strategic thinking.

Most other chapters were updated, improved, reorganized, and streamlined. The biggest changes occur in the chapters on the prisoners' dilemma (Chapter 10), collective action (Chapter 11), evolutionary games (Chapter 12), and voting (Chapter 15). We omitted the final chapter of the third edition (Markets and Competition) because in our experience almost no one used it. Teachers who want it can find it in the third edition.

We thank numerous readers of previous editions who provided comments and suggestions; they are thanked by name in the prefaces of those editions. The substance and writing in the book have been improved by the perceptive and constructive pieces of advice offered by faculty who have used the text in their courses and others who have read all or parts of the book in other

contexts. For the fourth edition, we have also had the added benefit of extensive comments from Christopher Maxwell (Boston College), Alex Brown (Texas A&M University), Jonathan Woon (University of Pittsburgh), Klaus Becker (Texas Tech University), Huanxing Yang (Ohio State University), Matthew Roelofs (Western Washington University), and Debashis Pal (University of Cincinnati). Thank you all.

Avinash Dixit
Susan Skeath
David Reiley

PART ONE

Introduction and General Principles

1

Basic Ideas and Examples

ALL INTRODUCTORY TEXTBOOKS begin by attempting to convince the student readers that the subject is of great importance in the world and therefore merits their attention. The physical sciences and engineering claim to be the basis of modern technology and therefore of modern life; the social sciences discuss big issues of governance—for example, democracy and taxation; the humanities claim that they revive your soul after it has been deadened by exposure to the physical and social sciences and to engineering. Where does the subject games of strategy, often called game theory, fit into this picture, and why should you study it?

We offer a practical motivation that is much more individual and probably closer to your personal concerns than most other subjects. You play games of strategy all the time: with your parents, siblings, friends, and enemies, and even with your professors. You have probably acquired a lot of instinctive expertise in playing such games, and we hope you will be able to connect what you have already learned to the discussion that follows. We will build on your experience, systematize it, and develop it to the point where you will be able to improve your strategic skills and use them more methodically. Opportunities for such uses will appear throughout your life; you will go on playing such games with your employers, employees, spouses, children, and even strangers.

Not that the subject lacks wider importance. Similar games are played in business, politics, diplomacy, and wars—in fact, whenever people interact to strike mutually agreeable deals or to resolve conflicts. Being able to recognize such games will enrich your understanding of the world around you and will make you a better participant in all its affairs. Understanding games of strategy will also have a more immediate payoff in your study of many other subjects. Economics and business courses already use a great deal of game-theoretic thinking. Political science, psychology, and philosophy are also using game theory to study interactions, as is biology, which has been importantly influenced by the concepts of evolutionary games and has in turn exported these ideas to economics. Psychology and philosophy also interact with the study of games of strategy. Game theory provides concepts and techniques of analysis for many disciplines, one might say all disciplines except those dealing with completely inanimate objects.

1 WHAT IS A GAME OF STRATEGY?

The word *game* may convey an impression that the subject is frivolous or unimportant in the larger scheme of things—that it deals with trivial pursuits such as gambling and sports when the world is full of weightier matters such as war and business and your education, career, and relationships. Actually, games of strategy are not “just a game”; all of these weighty matters are instances of games, and game theory helps us understand them all. But it will not hurt to start with game theory as applied to gambling or sports.

Most games include chance, skill, and strategy in varying proportions. Playing double or nothing on the toss of a coin is a game of pure chance, unless you have exceptional skill in doctoring or tossing coins. A hundred-yard dash is a game of pure skill, although some chance elements can creep in; for example, a runner may simply have a slightly off day for no clear reason.

Strategy is a skill of a different kind. In the context of sports, it is a part of the mental skill needed to play well; it is the calculation of how best to use your physical skill. For example, in tennis, you develop physical skill by practicing your serves (first serves hard and flat, second serves with spin or kick) and passing shots (hard, low, and accurate). The strategic skill is knowing where to put your serve (wide, or on the T) or passing shot (crosscourt, or down the line). In football, you develop such physical skills as blocking and tackling, running and catching, and throwing. Then the coach, knowing the physical skills of his own team and those of the opposing team, calls the plays that best exploit his team’s skills and the other team’s weaknesses. The coach’s calculation constitutes the strategy. The physical game of football is played on the gridiron by jocks; the strategic game is played in the offices and on the sidelines by coaches and by nerdy assistants.

A hundred-yard dash is a matter of exercising your physical skill as best you can; it offers no opportunities to observe and react to what other runners in the race are doing and therefore no scope for strategy. Longer races do entail strategy—whether you should lead to set the pace, how soon before the finish you should try to break away, and so on.

Strategic thinking is essentially about your interactions with others, as they do similar thinking at the same time and about the same situation. Your opponents in a marathon may try to frustrate or facilitate your attempts to lead, given what they think best suits their interests. Your opponent in tennis tries to guess where you will put your serve or passing shot; the opposing coach in football calls the play that will best counter what he thinks you will call. Of course, just as you must take into account what the other player is thinking, he is taking into account what you are thinking. Game theory is the analysis, or science, if you like, of such interactive decision making.

When you think carefully before you act—when you are aware of your objectives or preferences and of any limitations or constraints on your actions and choose your actions in a calculated way to do the best according to your own criteria—you are said to be behaving rationally. Game theory adds another dimension to rational behavior—namely, interaction with other equally rational decision makers. In other words, game theory is the science of rational behavior in interactive situations.

We do not claim that game theory will teach you the secrets of perfect play or ensure that you will never lose. For one thing, your opponent can read the same book, and both of you cannot win all the time. More importantly, many games are complex and subtle, and most actual situations include enough idiosyncratic or chance elements that game theory cannot hope to offer surefire recipes for action. What it does is provide some general principles for thinking about strategic interactions. You have to supplement these ideas and some methods of calculation with many details specific to your situation before you can devise a successful strategy for it. Good strategists mix the science of game theory with their own experience; one might say that game playing is as much art as science. We will develop the general ideas of the science but will also point out its limitations and tell you when the art is more important.

You may think that you have already acquired the art from your experience or instinct, but you will find the study of the science useful nonetheless. The science systematizes many general principles that are common to several contexts or applications. Without general principles, you would have to figure out from scratch each new situation that requires strategic thinking. That would be especially difficult to do in new areas of application—for example, if you learned your art by playing games against parents and siblings and must now practice strategy against business competitors. The general principles of game theory provide you with a ready reference point. With this foundation in place, you can proceed much more quickly and confidently to acquire and add the situation-specific features or elements of the art to your thinking and action.

2 SOME EXAMPLES AND STORIES OF STRATEGIC GAMES

With the aims announced in Section 1, we will begin by offering you some simple examples, many of them taken from situations that you have probably encountered in your own lives, where strategy is of the essence. In each case we will point out the crucial strategic principle. Each of these principles will be discussed more fully in a later chapter, and after each example we will tell you where the details can be found. But don't jump to them right away; for a while, just read all the examples to get a preliminary idea of the whole scope of strategy and of strategic games.

A. Which Passing Shot?

Tennis at its best consists of memorable duels between top players: John McEnroe versus Ivan Lendl, Pete Sampras versus Andre Agassi, and Martina Navratilova versus Chris Evert. Picture the 1983 U.S. Open final between Evert and Navratilova.[1] Navratilova at the net has just volleyed to Evert on the baseline. Evert is about to hit a passing shot. Should she go down the line or crosscourt? And should Navratilova expect a down-the-line shot and lean slightly that way or expect a crosscourt shot and lean the other way?

Conventional wisdom favors the down-the-line shot. The ball has a shorter distance to travel to the net, so the other player has less time to react. But this does not mean that Evert should use that shot all of the time. If she did, Navratilova would confidently come to expect it and prepare for it, and the shot would not be so successful. To improve the success of the down-the-line passing shot, Evert has to use the crosscourt shot often enough to keep Navratilova guessing on any single instance.

Similarly in football, with a yard to go on third down, a run up the middle is the percentage play—that is, the one used most often—but the offense must throw a pass occasionally in such situations "to keep the defense honest."

Thus, the most important general principle of such situations is not what Evert *should* do but what she *should not* do: she should not do the same thing all the time or systematically. If she did, then Navratilova would learn to cover that, and Evert's chances of success would fall.

Not doing any one thing systematically means more than not playing the same shot in every situation of this kind. Evert should not even mechanically switch back and forth between the two shots. Navratilova would spot and exploit

[1] Chris Evert won her first title at the U.S. Open in 1975. Navratilova claimed her first title in the 1983 final.

this *pattern* or indeed any other detectable system. Evert must make the choice on each particular occasion *at random* to prevent this guessing.

This general idea of "mixing one's plays" is well known, even to sports commentators on television. But there is more to the idea, and these further aspects require analysis in greater depth. Why is down-the-line the percentage shot? Should one play it 80% of the time or 90% or 99%? Does it make any difference if the occasion is particularly big; for example, does one throw that pass on third down in the regular season but not in the Super Bowl? In actual practice, just how does one mix one's plays? What happens when a third possibility (the lob) is introduced? We will examine and answer such questions in Chapter 7.

The movie *The Princess Bride* (1987) illustrates the same idea in the "battle of wits" between the hero (Westley) and a villain (Vizzini). Westley is to poison one of two wineglasses out of Vizzini's sight, and Vizzini is to decide who will drink from which glass. Vizzini goes through a number of convoluted arguments as to why Westley should poison one glass. But all of the arguments are innately contradictory, because Westley can anticipate Vizzini's logic and choose to put the poison in the other glass. Conversely, if Westley uses any specific logic or system to choose one glass, Vizzini can anticipate that and drink from the other glass, leaving Westley to drink from the poisoned one. Thus, Westley's strategy has to be random or unsystematic.

The scene illustrates something else as well. In the film, Vizzini loses the game and with it his life. But it turns out that Westley had poisoned both glasses; over the last several years, he had built up immunity to the poison. So Vizzini was actually playing the game under a fatal information disadvantage. Players can sometimes cope with such asymmetries of information; Chapters 8 and 13 examine when and how they can do so.

B. The GPA Rat Race

You are enrolled in a course that is graded on a curve. No matter how well you do in absolute terms, only 40% of the students will get As, and only 40% will get Bs. Therefore, you must work hard, not just in absolute terms, but relative to how hard your classmates (actually, "class enemies" seems a more fitting term in this context) work. All of you recognize this, and after the first lecture you hold an impromptu meeting in which all students agree not to work too hard. As weeks pass by, the temptation to get an edge on the rest of the class by working just that little bit harder becomes overwhelming. After all, the others are not able to observe your work in any detail; nor do they have any real hold over you. And the benefits of an improvement in your grade point average are substantial. So you hit the library more often and stay up a little longer.

The trouble is, everyone else is doing the same. Therefore, your grade is no better than it would have been if you and everyone else had abided by the agreement. The only difference is that all of you have spent more time working than you would have liked.

This is an example of the prisoners' dilemma.[2] In the original story, two suspects are being separately interrogated and invited to confess. One of them, say A, is told, "If the other suspect, B, does not confess, then you can cut a very good deal for yourself by confessing. But if B does confess, then you would do well to confess, too; otherwise the court will be especially tough on you. So you should confess no matter what the other does." B is told to confess, with the use of similar reasoning. Faced with this choice, both A and B confess. But it would have been better for both if neither had confessed, because the police had no really compelling evidence against them.

Your situation is similar. If the others slack off, then you can get a much better grade by working hard; if the others work hard, then you had better do the same or else you will get a very bad grade. You may even think that the label "prisoner" is very fitting for a group of students trapped in a required course.

Professors and schools have their own prisoners' dilemmas. Each professor can make his course look good or attractive by grading it slightly more liberally, and each school can place its students in better jobs or attract better applicants by grading all of its courses a little more liberally. Of course, when all do this, none has any advantage over the others; the only result is rampant grade inflation, which compresses the spectrum of grades and therefore makes it difficult to distinguish abilities.

People often think that in every game there must be a winner and a loser. The prisoners' dilemma is different—both or all players can come out losers. People play (and lose) such games every day, and the losses can range from minor inconveniences to potential disasters. Spectators at a sports event stand up to get a better view but, when all stand, no one has a better view than when they were all sitting. Superpowers acquire more weapons to get an edge over their rivals but, when both do so, the balance of power is unchanged; all that has happened is that both have spent economic resources that they could have used for better purposes, and the risk of accidental war has escalated. The magnitude of the potential cost of such games to all players makes it important to understand the ways in which mutually beneficial cooperation can be achieved and sustained. All of Chapter 10 deals with the study of this game.

Just as the prisoners' dilemma is potentially a lose-lose game, there are win-win games, too. International trade is an example; when each country produces more of what it can do relatively best, all share in the fruits of this international division of labor. But successful bargaining about the division of the pie is

[2] There is some disagreement regarding the appropriate grammatical placement of the apostrophe in the term *prisoners' dilemma*. Our placement acknowledges the facts that there must be at least two prisoners in order for there to be any dilemma at all and that the (at least two) prisoners therefore jointly possess the dilemma.

needed if the full potential of trade is to be realized. The same applies to many other bargaining situations. We will study these in Chapter 17.

C. "We Can't Take the Exam Because We Had a Flat Tire"

Here is a story, probably apocryphal, that circulates on the undergraduate e-mail networks; each of us has independently received it from our students:

> There were two friends taking chemistry at Duke. Both had done pretty well on all of the quizzes, the labs, and the midterm, so that going into the final they each had a solid A. They were so confident the weekend before the final that they decided to go to a party at the University of Virginia. The party was so good that they overslept all day Sunday, and got back too late to study for the chemistry final that was scheduled for Monday morning. Rather than take the final unprepared, they went to the professor with a sob story. They said they each had gone up to UVA and had planned to come back in good time to study for the final but had a flat tire on the way back. Because they didn't have a spare, they had spent most of the night looking for help. Now they were really too tired, so could they please have a makeup final the next day? The professor thought it over and agreed.
>
> The two studied all of Monday evening and came well prepared on Tuesday morning. The professor placed them in separate rooms and handed the test to each. The first question on the first page, worth 10 points, was very easy. Each of them wrote a good answer, and greatly relieved, turned the page. It had just one question, worth 90 points. It was: "Which tire?"

The story has two important strategic lessons for future partygoers. The first is to recognize that the professor may be an intelligent game player. He may suspect some trickery on the part of the students and may use some device to catch them. Given their excuse, the question was the likeliest such device. They should have foreseen it and prepared their answer in advance. This idea that one should look ahead to future moves in the game and then reason backward to calculate one's best current action is a very general principle of strategy, which we will elaborate on in Chapter 3. We will also use it, most notably, in Chapter 9.

But it may not be possible to foresee all such professorial countertricks; after all, professors have much more experience seeing through students' excuses than students have making up such excuses. If the two students in the story are unprepared, can they independently produce a mutually consistent lie? If each picks a tire at random, the chances are only 25% that the two will pick the same one. (Why?) Can they do better?

You may think that the front tire on the passenger side is the one most likely to suffer a flat, because a nail or a shard of glass is more likely to lie closer to that side of the road than to the middle, and the front tire on that side will